sponding stationary problem (1.2). In addition, if $\gamma(x, t) \equiv \gamma_\infty(x)$ for $t \geq T_\infty$, then the solution of the problem (1.1) settles relative to the solution of the problem (1.2) after a finite time T_∞.

COROLLARY 2. (Theorem on Asymptotic Stability of a Potential Flow). Under the conditions of Theorem 3.2 the potential flow $\mathbf{u}_\infty(x)$ is asymptotically stable relative to small perturbations which are potential at the entry of the region.

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INCLINATION ANGLES OF THE BOUNDARY IN MOVING LIQUID LAYERS

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In this paper creeping flows in thin layers of a viscous liquid are discussed with the capillary forces taken into account, and solutions describing the inclination angles of the boundary are found. The contact angle of a liquid on a solid surface in the static state is expressed in terms of the specific surface energies. Upon movement of the liquid the contact angle (dynamic) differs from the static value. A very thin "precursor" film can be observed in front of the liquid mass which is spreading over the solid surface [1, 2]. There are indications to the effect that the value of the dynamic contact angle depends on the viscous forces [3].

1. Established Flow of a Liquid Layer over a Dry Surface and the Contact Angles. The pressure p inside a thin liquid layer on a flat solid surface differs from the pressure p_0 in the gas by the amount of the capillary differential $p = p_0 - \sigma \partial^2 h/\partial x^2$ (σ is the surface tension coefficient; x is the coordinate along the layer; and h is the thickness of the layer).

The equation of motion of the layer in the case of small Reynolds numbers under the action of capillary forces can be written with the help of the hydrodynamical theory of lubrication as

$$\frac{\partial}{\partial x}\left(\frac{\sigma}{3\mu} h^3 \frac{\partial^3 h}{\partial x^3}\right) = -\frac{\partial h}{\partial t}.$$

Non-steady-state solutions of this equation are investigated in the linear approximation in [4]. Let us consider steady-state solutions in the nonlinear formulation. For a steady-state wave

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$$h = h(x - vt),$$

and the indicated equation is simplified to

$$(\sigma/3\mu)h^3 d^3h/d\xi^3 = vh + K,\ \xi = x - vt,\ K = \mathrm{const}. \tag{1.1}$$

The constant K = 0 if the thickness of the layer h = 0 for $\xi > \xi_0$. This situation corresponds to the problem in which the layer flows (v > 0) onto a dry surface or runs off the surface (v < 0).

Let us assign the inclination angle of the free boundary in the case of small thickness h_m, and let us investigate the solutions for which the free boundary is close to rectilinear

$$\begin{gathered} dh/d\xi = -\alpha_m,\ h = h_m, \\ d^2h/d\xi^2 \to 0,\ h \to \infty \end{gathered} \tag{1.2}$$

in the range of large thicknesses h.

We will select the quantity h_m extremely small from the point of view of the applicability of the hydrodynamical description. Then it is possible to suppose that the microscopic angle α_m is close to the static contact wetting angle or the static hysteresis wetting angle. The problem formulated has a unique solution if v > 0.

The meaning of the problem (1.1) and (1.2) consists of the fact that it describes the surface profile in some small region close to the edge of the liquid volume for different problems of the motion of macroscopic liquid volumes. For example, in the case of the motion of a liquid adjoining a gas in a capillary the problem (1.1) and (1.2) corresponds to a range of thicknesses small in comparison with the diameter of the capillary. The complete solution can be found by the method of splicing.

The solution is sought in the form

$$dh/d\xi = (3\mu v/\sigma)^{1/3}u(s),\ s = \ln(h_m/h). \tag{1.3}$$

The problem (1.1) and (1.2) is reduced to the form

$$\begin{gathered} u''u^2 + u'u^2 + u'^2u = 1, \\ u = -\alpha_m M^{-1/3},\ s = 0;\ M = 3\mu v/\sigma, \\ u'ue^s \to 0,\ s \to -\infty. \end{gathered} \tag{1.4}$$

Equation (1.4) is not changed upon a shift along s. Therefore, the solution which satisfies the condition at infinity has the form

$$u = u(z),\ z = s + C. \tag{1.5}$$

The constant C is determined from the equation

$$u(C) = -\alpha_m M^{-1/3}. \tag{1.6}$$

We will determine the asymptotic expansion of u(z) as $|z| \to \infty$. To this end it is possible to use an iterative process based on the fact that the principal term on the left-hand side of Eq. (1.4) as $|z| \to \infty$ in the case of the specified condition at infinity is $u'u^2$:

$$u'_{i+1}u^2_{i+1} + u''_i u^2_i + u'^2_i u_i = 1;\ u_0 = 0,\ i = 0, 1, 2, \ldots$$

Taking account of the three approximations, the asymptotic representation has the form

$$u(z) = (3z)^{1/3}\left(1 + \frac{\ln|z|}{9z} + \frac{\ln|z| - 1/3\ln^2|z| - 4}{27z^2} + \ldots\right). \tag{1.7}$$

Equation (1.7) is applicable when $|z| > 1$, and its first term describes the run of u(z) at $z \sim 0$ in a qualitatively correct way.

Proceeding from (1.7), we will determine the form of the solution for different values of $\alpha_m M^{-1/3}$. First of all, let us consider the case of the flow of a liquid onto a solid surface, v > 0. In this case u < 0. The root C of Eq. (1.6) varies from $-\infty$ to the value $C \sim 0$ when $\alpha_m M^{-1/3}$ changes from $+\infty$ to 0. The difference of the solution u(s) from the first term of the expansion (1.7) is noticeably revealed only when C > −1 and only for the values $s \sim 0$. For s < −1 this difference is always insignificant, and as $s \to -\infty$ it tends to zero. Thus the detailed behavior of u(z) for −1 < z < 0 is not important for the determination of u(s) at $|s| \gg 1$. It is sufficient to use the first term of the representation (1.7) for the approximate determination of the root of (1.6).

In the case of a liquid running off, when $v < 0$, the situation is more complicated. The quantity $u > 0$. The desired solution $u(s)$ is obtained from $u(z)$, $z > 0$, by a shift. It is clear that a solution exists only in the finite region $-C < s < 0$. The quantity $u \sim 0$ at $s = -C$. In connection with this fact the condition that $|u'u|e^s$ decrease as $|s|$ increases is meaningfully discussed only on a finite interval. This interval should be sufficiently large, and consequently it is necessary that $C \gg 1$.

The approximate solution determined according to the first term of the asymptote (1.7) has the form

$$u = (3s - \alpha_m^3/M)^{1/3};\ C = -\alpha_m^3/(3M). \tag{1.8}$$

When $\alpha_m^3 \gg 3M > 0$, the solution is asymptotically exact for all values of s. When $\alpha_m^3 \lessdot 3|M|$, the solution gives only a qualitative picture for the values $s \sim 0$. However, it is possible to show that a more exact determination of $u(s)$ at $s \sim 0$ has meaning.

Let $\alpha_m^3 \lessdot 3|M|$. In this connection it is possible to discuss only flowing on, since $C \sim 1$ in the problem of running off, and the inclination angle of the free boundary becomes zero at a quite small height $h \sim 2h_m$.

The function $u(s)$ varies significantly in the region $-1 \lessdot s < 0$, i.e., the inclination angle of the free boundary varies sharply in the height range $h \sim h_m$ as the height h changes. This fact means that there is a sharp jog on the free boundary – a microscopic "bump". The "bump" has a thickness of the order of a few molecules. With the appearance of the "bump" it is possible to determine the microscopic contact angle only as to its order of magnitude, since the inclination of the free boundary near the microscopic "bump" varies greatly. Of course, only a qualitative description of the "bump" is possible because this is essentially a microscopic phenomenon.

In dimensional notation the condition for the appearance of the "bump" in the case of flowing on has the form

$$9\mu v/\sigma \gtrsim \alpha_m^3. \tag{1.9}$$

The formula for the inclination angle of the free boundary has, on the basis of (1.8) and (1.3), the form

$$\alpha = \left(\alpha_m^3 + \frac{9v\mu}{\sigma} \ln \frac{h}{h_m}\right)^{1/3},\ \alpha = -\frac{dh}{dx}, \tag{1.10}$$

as a function of its height h above the solid surface.

According to the definition given above, the quantity $h_m \sim 10^{-7}$–10^{-6} cm. One has $\ln(h/h_m) \sim 12$ for macroscopic thicknesses $h \sim 10^{-2}$–10^{-1} cm of the layer. As is evident from (1.10), the angle α varies weakly with height in the macroscopic region. Due to this fact it can be shown that a distinctly expressed contact angle occurs.

According to Eq. (1.10), a nonzero macroscopic contact angle α is exhibited on account of the motion ($v > 0$) even if the equilibrium wetting contact angle is equal to zero, $\alpha_m \sim 0$. We note that according to (1.9) there is always a "bump" on the free boundary in the case of complete wetting.

In the case of running off of liquid from a solid surface, when $v < 0$, the angle α drops off with height and becomes zero at some rather large height. Upon a continuation of the solution the quantity h passes through a minimum, and as $x \to -\infty$, the solution is close to a parabola, $h'' > 0$. It has been shown above that the macro-

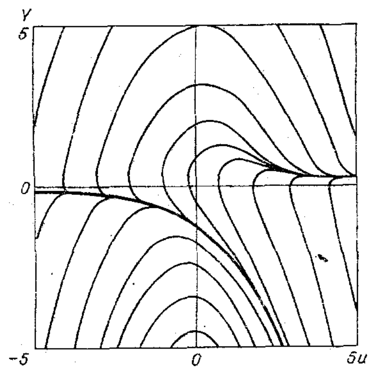

Fig. 1

scopic contact angle in the case of running off exists only in a limited range of heights h. The conditions for its existence are

$$9\,|v|\,\mu/\sigma \ll \alpha_m^3,\ h \ll h_m \exp\left(\alpha_m^3\sigma/9\mu\,|v|\right).$$

2. Splicing the Solution with the Meniscus. The asymptotic solution (1.7) permits effectively solving the problem when the condition $h'' \to \text{const} > 0$ as $h \to \infty$ is set and the constant quantity is small. A profile of constant curvature ($h'' = \text{const}$ in the thin-layer approximation) is called a meniscus. The transition from (1.7) to a meniscus of small curvature can be traced out by first investigating the exact solutions of the differential equation (1.4). The substitution $Y = u\,du/ds$ reduces it to the form $dY/du = 1/Y - u$.

The integral curves of this equation are given in Fig. 1. When $|u| \gg 1$, the curves are similar to portions of the hyperbolas $Y = 1/u$, which corresponds to (1.7). As $|u|$ increases, the curves depart from the u axis, and the parabola $h'' = \text{const}$, i.e., the meniscus, corresponds to the limit $|Y| \to \infty$. The solution $h'' \to 0$ as $s \to -\infty$ is the envelope of the family of solutions.

The substitution $w = u^2$ reduces Eq. (1.4) to the form

$$d^2w/dz^2 + dw/dz = -2\,\mathrm{sgn}\,(v)/\sqrt{w},\quad w = u^2. \tag{2.1}$$

We further consider the case of flowing on, $v > 0$. It is possible to show that in the case of running off ($v < 0$) the solution of Sec. 1 is not compatible with a meniscus.

We will seek a solution in the vicinity of the asymptotic one, taking account of the two terms in (1.7),

$$w = (3z)^{2/3}(1 + (2/9)z^{-1}\ln|z|) + w_1,\quad |w_1| \ll w. \tag{2.2}$$

The function w_1 satisfies the equation

$$w_1'' + w_1' + \frac{1}{3}z^{-1}w_1\left(1 - \frac{1}{3}z^{-1}\ln|z|\right) = 0. \tag{2.3}$$

The solution of this equation which increases as $z \to -\infty$ is represented in the form

$$w_1 = c_1|z|^{1/3}\left(1 + \frac{3+\ln|z|}{9z}\right)e^{-z} + \ldots,\ |z| \gg 1, \tag{2.4}$$

where c_1 is an arbitrary constant. It is possible to find the quantity c_1 after splicing.

As $z \to -\infty$, the solution of (2.1) which corresponds to $h'' > 0$ has the form

$$w = a_1e^{-z} + a_2 + w_2. \tag{2.5}$$

Now $w_2 \to 0$ as $z \to -\infty$. Let us assume the quantity a_1 to be known. It is necessary for the existence of the region of applicability of (2.2) that $a_1 \ll 1$. This fact is a fundamental restriction of the entire analysis. We note that when $a_1 \gtrsim 1$ there exists no region in which the inclination angle of the free boundary varies weakly with the height. The condition $a_1 \ll 1$ corresponds to the fact that the occurrence of a dynamic contact angle is possible.

The splicing of (2.2) and (2.5) permits determining immediately in the first approximation the quantity a_2 and the splicing region $z \sim z_*$:

$$a_1e^{-z_*} = (3z_*)^{2/3},\ a_2 \approx (3z_*)^{2/3}\left(1 + \frac{2}{9}z_*^{-1}\ln|z_*|\right) \gg 1. \tag{2.6}$$

Let us find w_2 in (2.5) in order to accomplish the asymptotic splicing. The function w_2 satisfies the equation

$$d^2w_2/dz^2 + dw_2/dz = -2/\sqrt{a_1e^{-z} + a_2} \tag{2.7}$$

with an accuracy out to small quantities of the order of $1/a_2^{3/2}$.

The solution of this equation, which vanishes as $z \to -\infty$, has the form

$$w_2 = \frac{1}{\sqrt{a_2}}\left[\left(\frac{a_1}{a_2}e^{-z} + 2\right)\left(\ln\frac{a_1}{a_2} - z - 2\ln\left(1 + \sqrt{\frac{a_1}{a_2}e^{-z} + 1}\right)\right) + 2\sqrt{\frac{a_1}{a_2}e^{-z} + 1}\right]. \tag{2.8}$$

The representation (2.5) and (2.8) is applicable when $|w_2| \ll w$. In this connection the region of its applicability overlaps the region of applicability of the representation (2.2) and (2.4). Let us consider values of z such that

$$a_1 e^{-z} \ll a_2, \quad |z - z_*| \ll |z_*|.$$

Using one term of the expansion into a series in the small quantity $a_1/a_2 e^z$ and two terms of the expansion in $(z - z_*)$ in Eq. (2.8), as well as two terms of the expansion in $(z - z_*)$ in the first term in (2.2), we require agreement of the representations (2.5), (2.8) and (2.2), (2.4). At the same time we find the quantity c_1 in (2.4) and obtain an equation for a_2:

$$a_2 + (2 - 4\ln 2)/\sqrt{a_2} = (3z_*)^{2/3}\left(1 + \frac{2}{9} z_*^{-1} \ln|z_*|\right). \tag{2.9}$$

Within the framework of the accuracy with which the splicing was performed, the function $a_2(a_1)$ can be found asymptotically from (2.9) and (2.6) in the form

$$a_2 = (3\ln(1/a_1) + \ln\ln(1/a_1) + 6\ln 2 + 2\ln 3 - 3)^{2/3}. \tag{2.10}$$

According to Eq. (2.5) [with (1.3) taken into account], the quantity $\sqrt{a_2}$ is proportional to the angle α_0, which forms a meniscus with the solid surface, if one formally continues it into the region $h \to 0$. The quantity a_1 is expressed in terms of the radius of curvature of the meniscus R_0 as

$$a_1 = 2e^{C}(h_m/R_0)(\sigma/3\mu v)^{2/3}; \quad \alpha_0 = (3\mu v/\sigma)^{1/3}\sqrt{a_2}. \tag{2.11}$$

The quantity C is determined in agreement with (1.5).

We note that the characteristic height h_s at which the transition occurs from the solution with an inclination angle, which slowly varies with respect to height, to the meniscus, on which the cosine of the angle varies proportionally to height, is equal to $h_s \approx 1/2\alpha_0^2 R_0$, as follows from (2.6) and (1.5) [with (1.3) taken into account].

The condition of applicability of (2.10) is $a_1 \ll 1$ in (2.11). If a_1 is not very small, then the rougher approximation (2.6) may give higher accuracy than (2.10), as usually happens in asymptotic solutions.

3. Flowing of a Liquid Layer onto a Surface Covered by a Thin Layer of Liquid. Let a liquid be moving over a solid surface onto which a uniform layer of the same liquid of thickness h_∞ is deposited in advance. Then the constant $K = -vh_\infty$ in (1.1) and the first condition of (1.2) is replaced by the condition that in the limit $x \to \infty$ the layer becomes a motionless uniform one,

$$\begin{gathered}(\sigma/3\mu j)h^3 d^3h/dx^3 = v(h - h_\infty),\\ h'' \to 0,\ h \to \infty\ (x \to -\infty),\\ h \to h_\infty,\ x \to \infty.\end{gathered} \tag{3.1}$$

The coefficient j in Eq. (3.1) is introduced in connection with the fact that the problem under discussion refers not only to the motion of a pure liquid over a film on a solid surface ($j = 1$) but is also valid when the free boundary is slowed down due to the effect of surface-active materials ($j = 4$). In the case $j = 8$ we obtain the problem of the motion of a liquid over a free film of thickness h, both of whose boundaries are slowed down under the action of surface-active materials. If in the latter case we convert, with the help of a Galilean transformation, to a system of coordinates in which the liquid in the region $h \to \infty$ is motionless, then we obtain the problem of the flow of a free liquid film into a meniscus (the Plateau boundary) of small curvature.

Let us consider the case of $v > 0$. It is possible to show that when $v < 0$ there is no solution of the boundary-value problem (3.1). Having applied the substitution of variables (1.3) with $h_m = h_\infty$, we obtain

$$\begin{gathered}u''u^2 + u'u^2 + u'^2u = 1 - e^s,\\ u'ue^s \to 0,\ s \to -\infty,\\ u = 0,\ s = 0.\end{gathered} \tag{3.2}$$

in place of (3.1).

If one neglects the exponentially small terms as $s \to -\infty$, then we obtain from (3.2) the fact that the asymptote of u as $s \to -\infty$ is the same as that of the solution of Eq. (1.4), i.e., determined by Eqs. (1.5) and (1.7). The constant C in (1.5) is determined by the condition $u = 0$ at $s = 0$. The constant C is found by means of numerical calculations. It is numerically more convenient to solve the third-order equation (3.1) than (3.2), since the integral curve of (3.2) encloses in a small neighborhood the origin of coordinates in the u, s plane. Numerical calculations have permitted finding C = 0.61. At the same time it is possible to obtain from (1.3), (1.5), and (1.7), taking two terms into account, an asymptotic expression for the inclination angle of the free boundary:

$$\alpha = (9\,\mu v/\sigma)^{1/3}[\ln (h/h_\infty) - 0.61 - (1/3)\ln (\ln (h/h_\infty) - 0.61)]^{1/3}. \tag{3.3}$$

As a comparison with the exact numerical solution shows, the condition of applicability of the asymptotic Eq. (3.3) is $h > 3h_\infty$. At $h = 3h_\infty$ the error of this formula is about 10%. At $h \geq 8h_\infty$ the error of Eq. (3.3) is less than 2% and decreases with an increase in h.

Until now the asymptotic behavior of the inclination angle α was not known. It has not proven successful to determine the nature of the limiting behavior of α from numerical calculations. The incorrect result $\alpha \to$ const as $h \to \infty$ has been obtained in [5] from numerical calculations. Therefore, the final formula of [5] is in error. A point of view similar to [5] has been expressed in [6], where the existence has been assumed of a limiting angle of the meniscus with the surface as the curvature of the meniscus tends to zero. Actually, as is evident from Eq. (3.3), no limiting inclination angle exists as $h \to \infty$.

Let the liquid force the gas out of a capillary, whereby the surface of the capillary is covered by a film whose thickness is much less than its diameter. For small angles α_0 the dependence of the angle on velocity is given by Eqs. (2.10) and (2.11), where by h_m it is necessary to understand h_∞, and the quantity C = 0.61. If the value of a_1 is insufficiently small, then it is better to use instead of (2.10) the analytically cruder formula which follows from (2.6). If $a_1 \gtrsim 1$, then in general there is no contact angle. In this case continuation of the meniscus as far as the solid boundary h = 0 and up to the height of the film $h = h_\infty$ gives significantly different values of α_0. Consequently, the contact angle has an asymptotic meaning, and it is possible to determine it in the limit of small values of the parameter a_1.

4. The Effect of van der Waals Forces on the Dynamic Contact Angle. The van der Waals forces can be important on a small scale in the case of a small curvature of the free boundary. The problem of the flow of a viscous film is discussed in [7] with the neglect of capillary forces. The taking of van der Waals forces into account can be important for the case of a wetting liquid with a zero equilibrium contact angle if the velocity of the motion is small.

For small inclination angles of the free boundary the effective pressure inside the layer obeys the equation

$$p = p_0 - \sigma d^2h/dx^2 + A/6\pi h^3.$$

The constant A takes account of the van der Waals forces (for example, see [8-10]). $A < 0$ for the case of complete wetting. With the van der Waals forces taken into account, Eq. (1.1) is complicated somewhat:

$$\frac{h^3}{3\mu}\frac{d}{dx}\left(\sigma\frac{d^2h}{dx^2} - \frac{A}{6\pi h^3}\right) = hv. \tag{4.1}$$

Condition (1.2) remains in force as $h \to \infty$. It is possible in the region of small thicknesses to require $h' \to 0$ as $h \to 0$, formally extending the solution right down to h = 0.

In the dimensionless symbols ζ and y

$$x = h_m(\sigma/3\mu v)^{1/3}\zeta, \quad h = h_m y \tag{4.2}$$

Eq. (4.1) will take the form

$$y^3\frac{d^3y}{d\zeta^3} - \frac{\beta^2}{y}\frac{dy}{d\zeta} - y = 0, \quad \beta^2 = -\frac{A}{2\pi\sigma}\left(\frac{\sigma}{3\mu v}\right)^{2/3}\frac{1}{h_m^2}. \tag{4.3}$$

The case $\beta \gg 1$ is interesting. In the range of small y it is possible to seek $dy/d\zeta$ in the form of a series in β^{-2}, assuming the term y^3y''' in (4.3) to be small,

$$\frac{dy}{d\zeta} = -\frac{1}{\beta^2}\left(1 + \frac{6y^6}{\beta^6} + \ldots\right)y^2, \quad y \ll \beta \tag{4.4}$$

It is possible to show that the asymptotic series in β^{-2} is divergent. Taking more than two terms into account has no practical meaning. In the region of large values of y the form of the solution agrees with the investigations in Sec. 1:

$$y' = -3^{1/3}(\ln y - (1/3)\ln\ln y - c_1)^{1/3}, \quad y \gg \beta. \tag{4.5}$$

Comparison of (4.5) and (1.10) with $y \sim \beta$ permits evaluation of the constant in (4.5). The quantity $c_1 \sim \ln\beta$. Consequently, the height h at which the inclination angle of the free boundary is close to zero is $\sim h_m\beta$ with $\beta \gg 1$ instead of h_m with $\beta = 0$.

Using Eq. (1.10), it is possible to show that for the possible values of the constant A [8] the value $\beta \gg 1$ only when $\alpha_0 \ll 1$, where α_0 is the macroscopic contact angle. If the angle α_0 is small and the value of A is sufficiently large, then one should substitute into Eq. (1.10) in place of the quantity h_m, defined to be a few molecular diameters, the quantity

$$h_m \approx \sqrt{|A|/2\pi\sigma}(\sigma/3\mu v)^{1/3} \tag{4.6}$$

which is $\sim\beta$ times larger.

According to Eqs. (4.2) and (4.4) and the definition of β^2 in (4.3), for $h \ll h_m$

$$h = |A|/6\pi\mu v(x - x_0). \tag{4.7}$$

The so-called precursor film, which moves in front of a spreading liquid, has been observed in the case of a very low wetting rate in experiments [1]. Up until now this phenomenon has remained unexplained [2]. Analyzing the two profiles of the precursor film given in Fig. 4 of [1], it is possible to determine that they are described, within the limits of error of the experiments, by the curves $h = c(t)/(x - x_0)$, which corresponds to (4.7). We find from the data of [1] for the instant t = 18 h after the start of the spreading that the quantity $c \sim 4 \cdot 10^{-8}$ cm², while the velocity $v \sim 5 \cdot 10^{-7}$ cm. If one takes account of the values $\mu = 0.27$ P and $\sigma = 27.6$ dyn/cm [1], then we determine from (4.7) the reasonable value $|A| \sim 10^{-13}$ erg, which is in agreement with the typical values of the constant A [8]. At the same time the quantity $h_m \sim 10^{-5}$ cm, according to (4.6). The length of the precursor film, determined by the condition $h_m > h > 10^{-7}$ cm, is equal to ~ 0.3 cm.

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